Generalisation of Recursive Doubling for AllReduce

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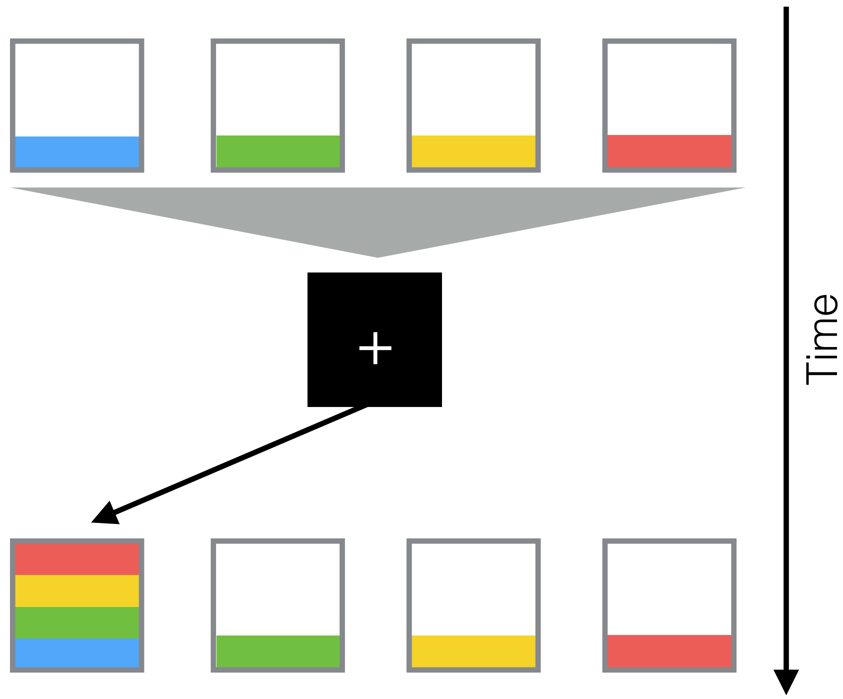
informatics

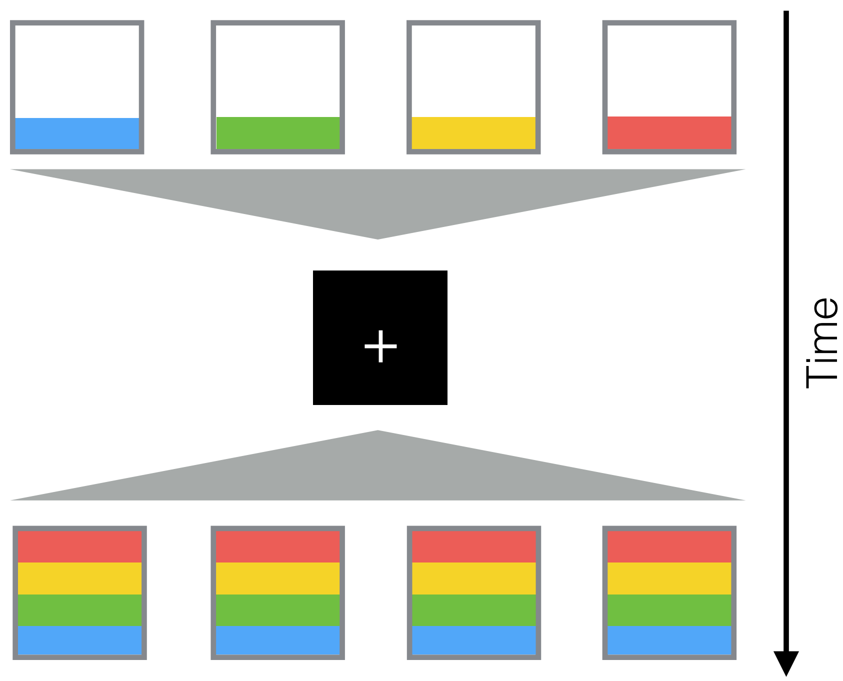
Setting the stage

- AllReduce is one of the most important MPI collectives
- AllReduce is a core dependency of iterative solvers

Operation

- **Reduction** with an operator
  - max, min, +, *, etc…
- Over networked **distributed memory** nodes
  - [2, inf)
- **All** processes need the same answer

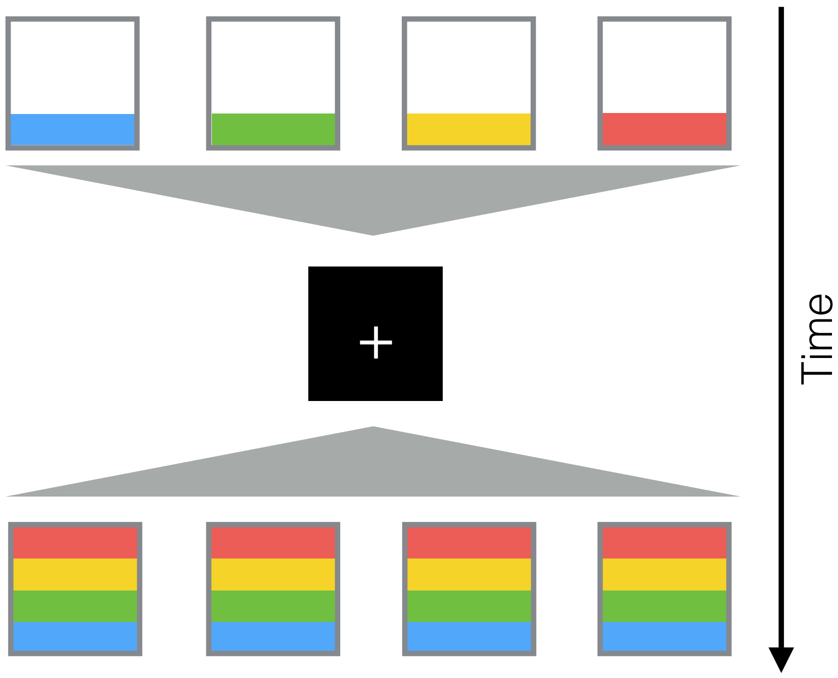


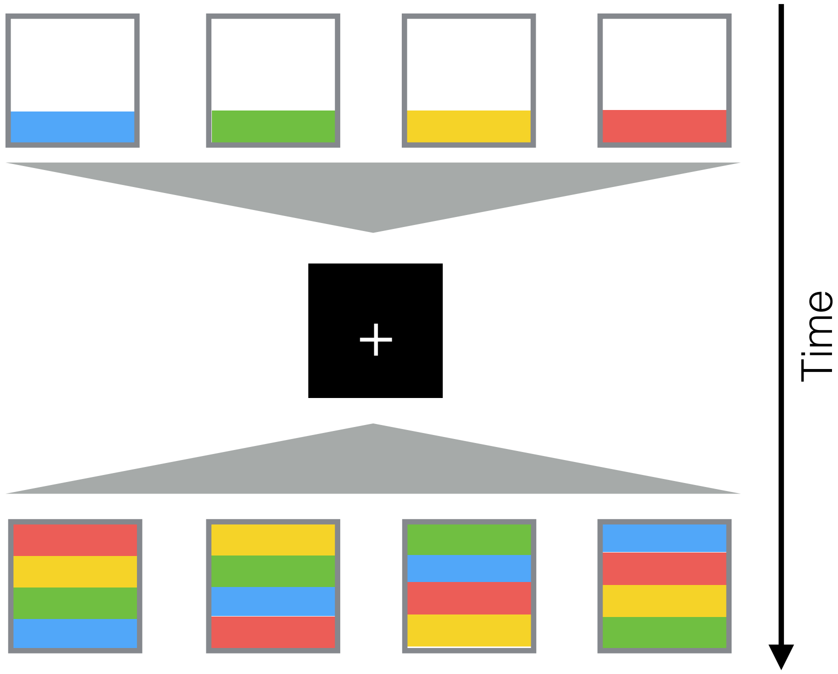


Constraints

- Consistent results required **across executions**
- Consistent results **across processes**
- **Ordering** of non-associative operators

$$(a+b)+c \neq a+(b+c)$$

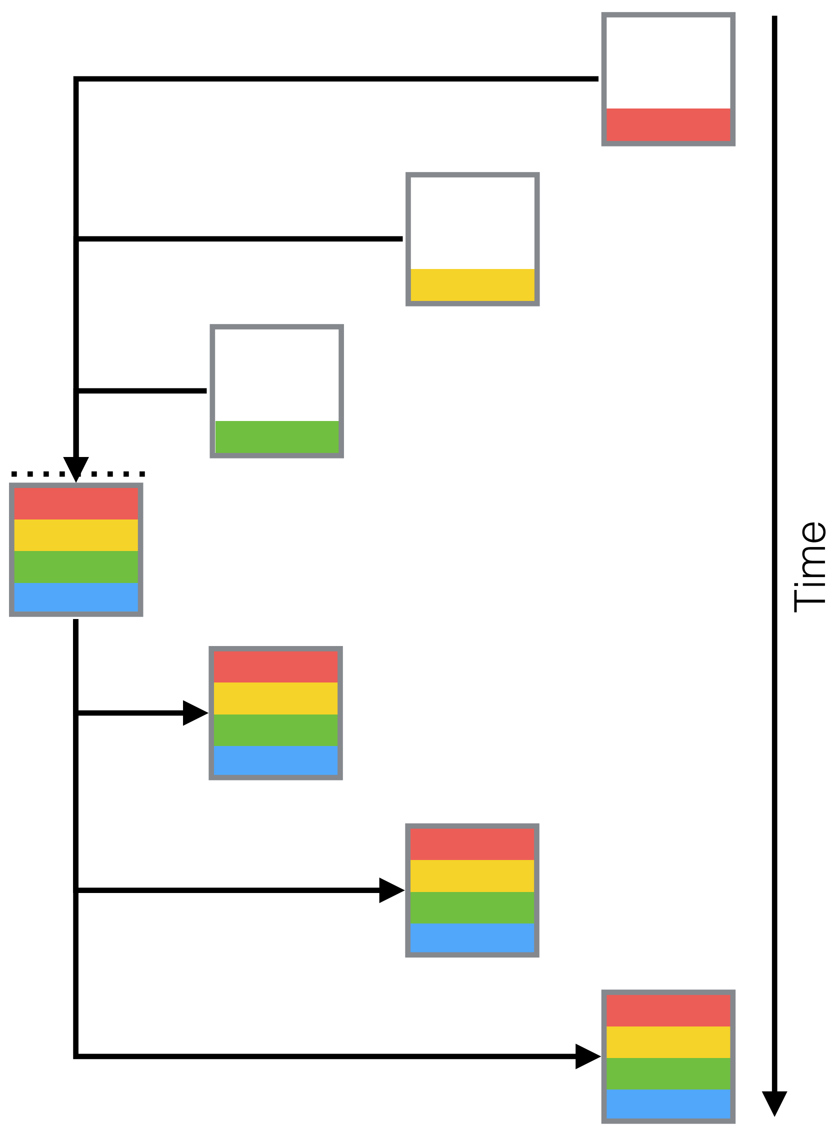




Simple Ways

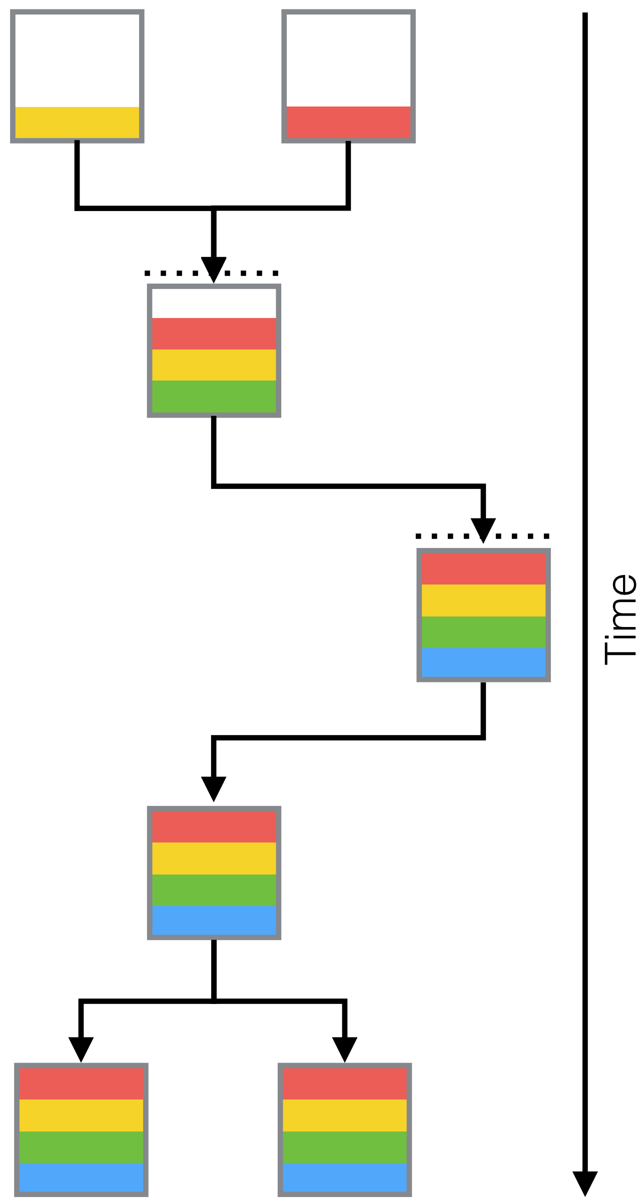
Linear Reduce & Broadcast

Time: $O(N)$



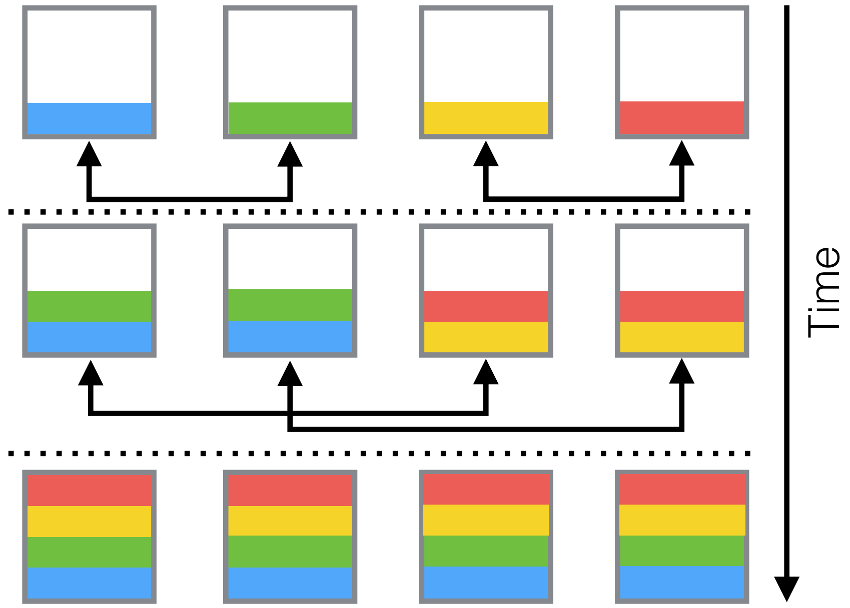
Tree Reduce & Broadcast

Time: $2\times O(\log_2 N)$

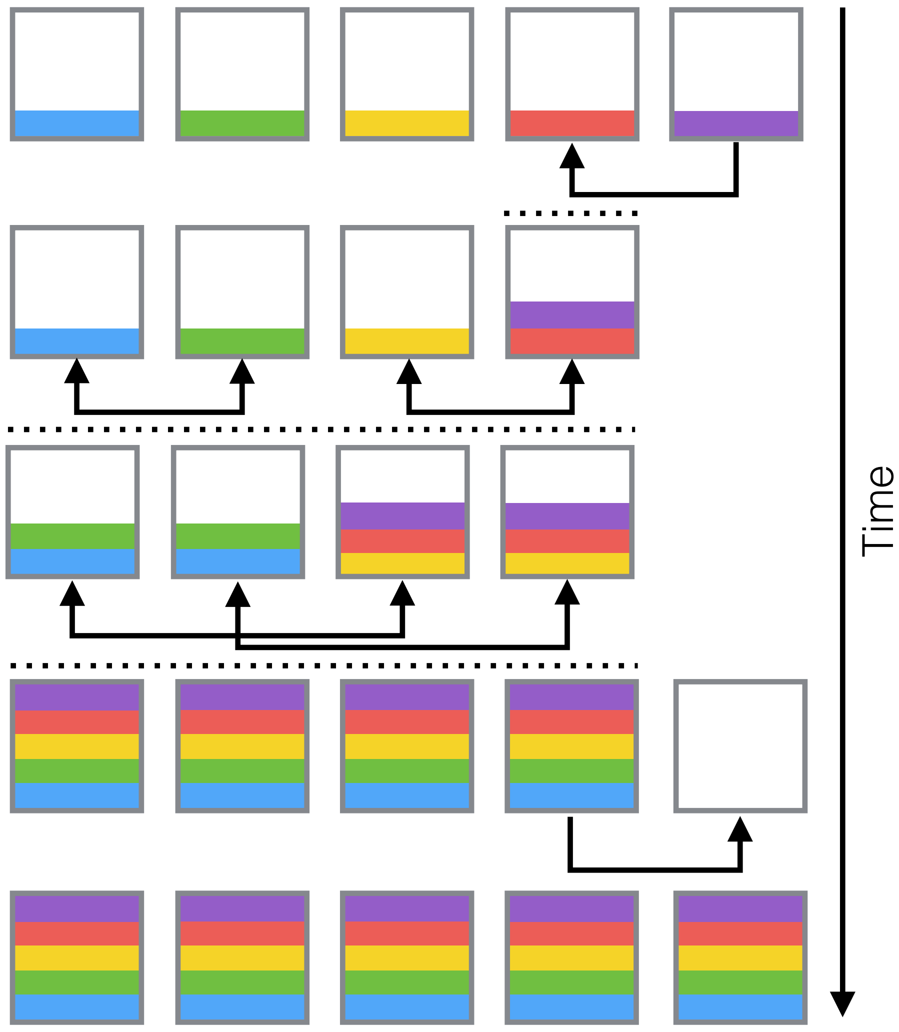


Butterfly Pattern

- Pairwise Exchange using Recursive Doubling
  - Time: $O(\log_2 N)$, requires $N = 2^k$
  - There is a fix, but costs 2 stages!

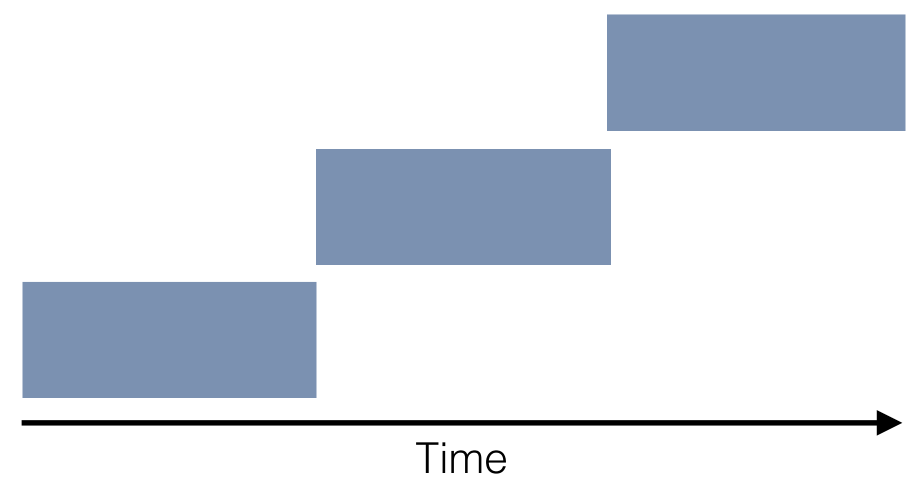


$$stages = \log_2 4 = 2$$



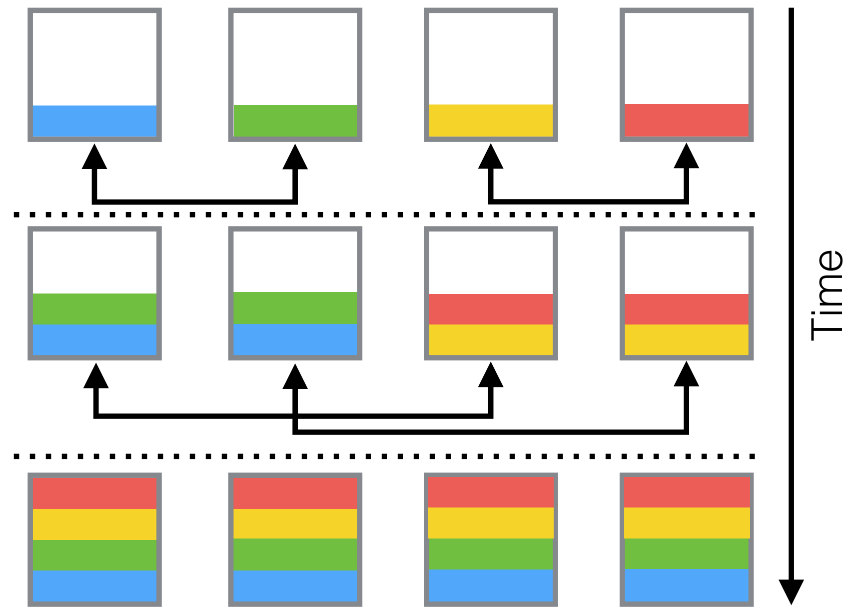
Modelling

- Simple cost model
  - For **small messages**
  - **Latency bound** messages

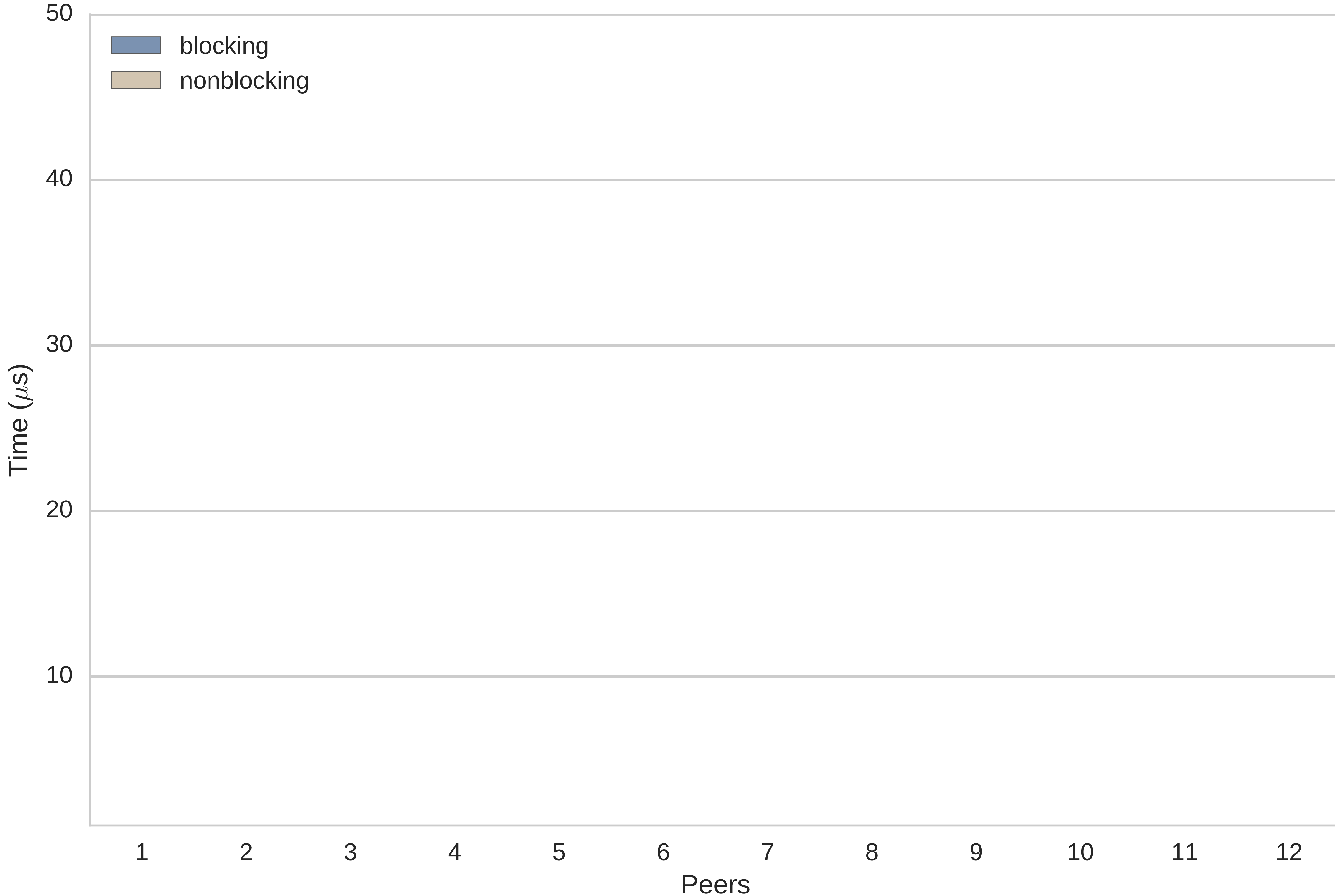


$$\alpha + \cancel{\beta n + \gamma n} \qquad n = 0$$

$$O(\log_2 N) \rightarrow \alpha \log_2 N$$



DMAPP Multicast Sends



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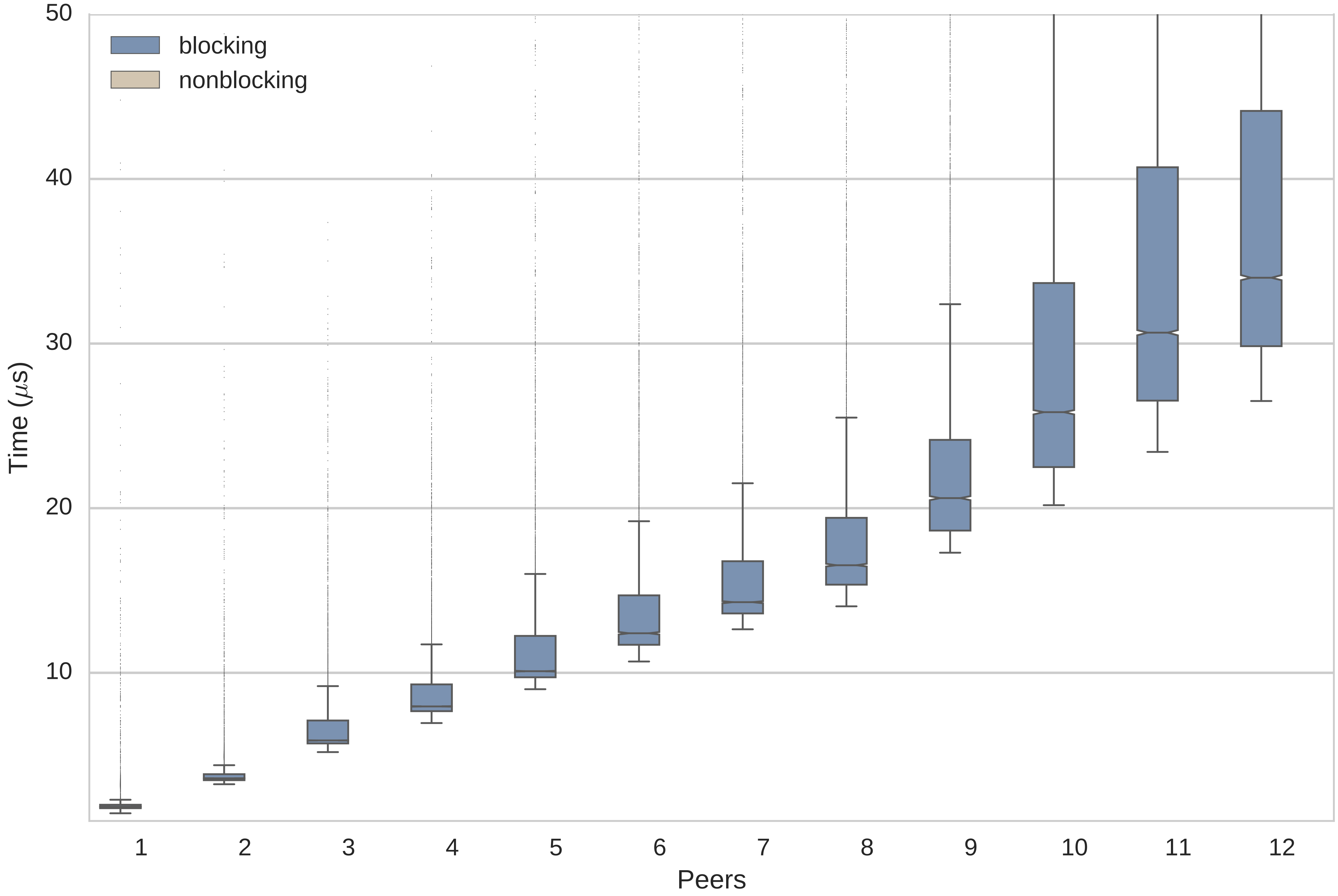

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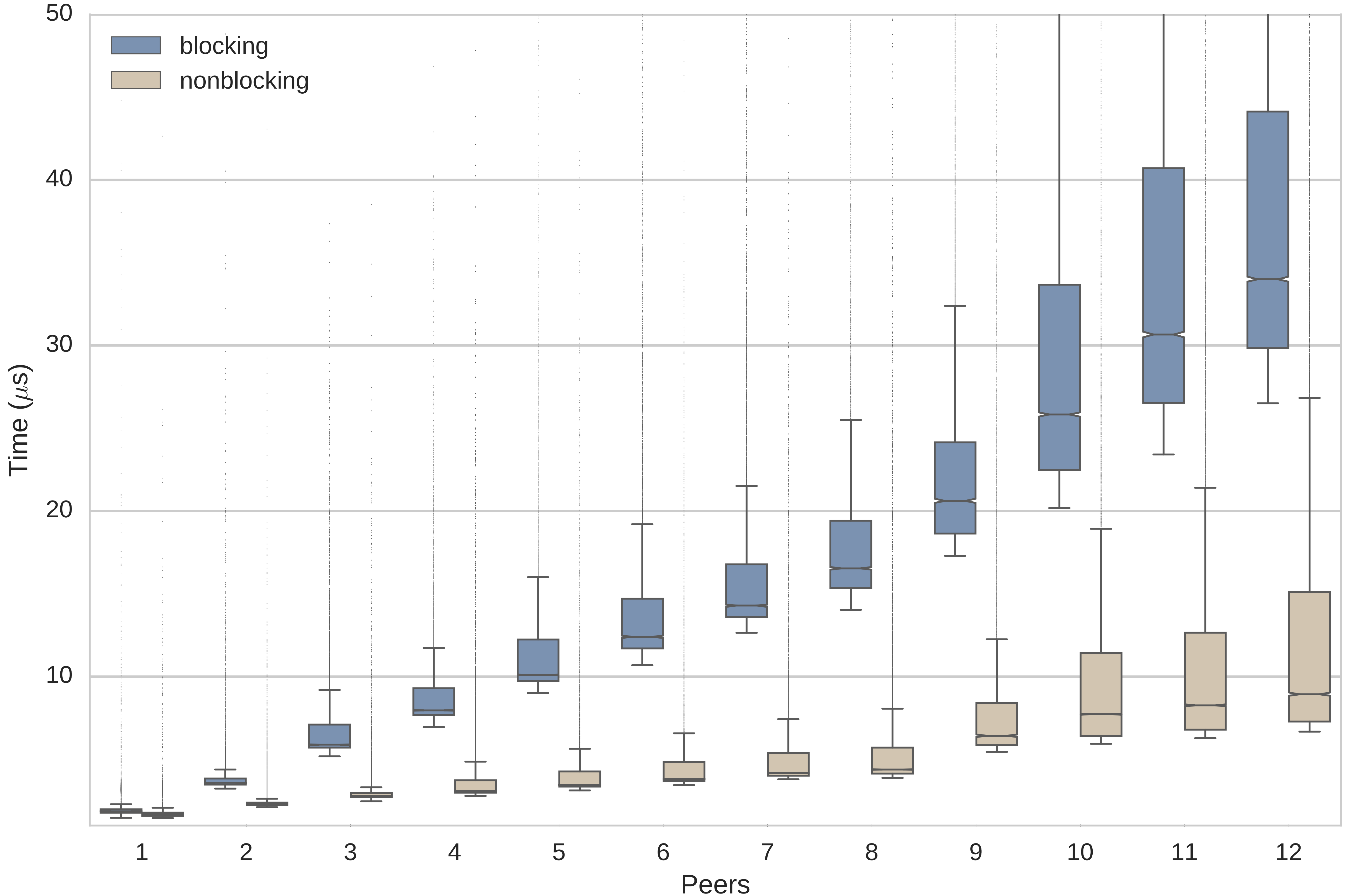
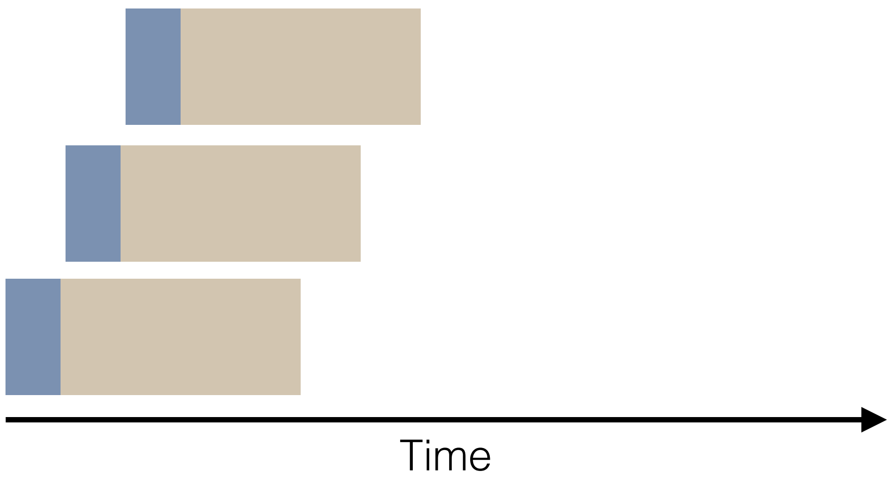


Modelling

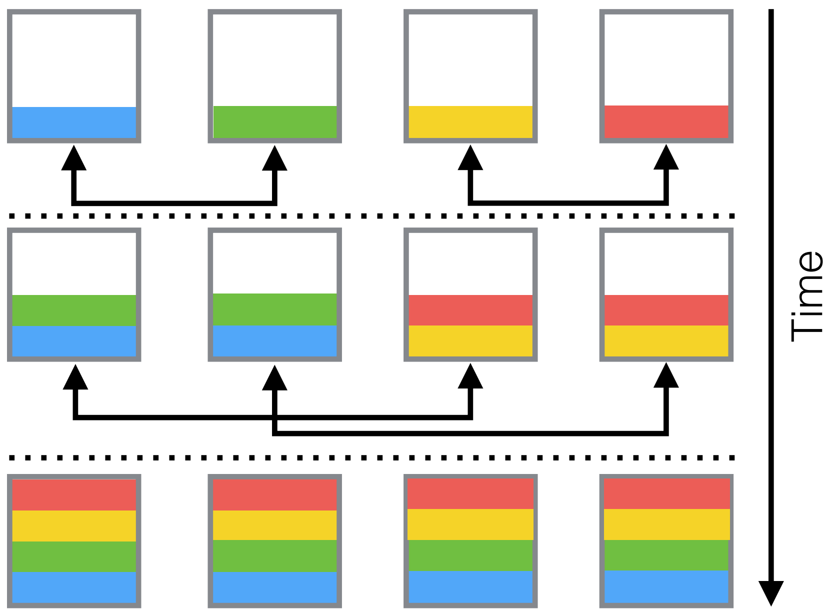
- Pipelining cost model
  - Host only sees partial cost
  - Offloaded to network
  - Results in **cheap multicasting**



$$\alpha = \alpha_p + \alpha_r$$

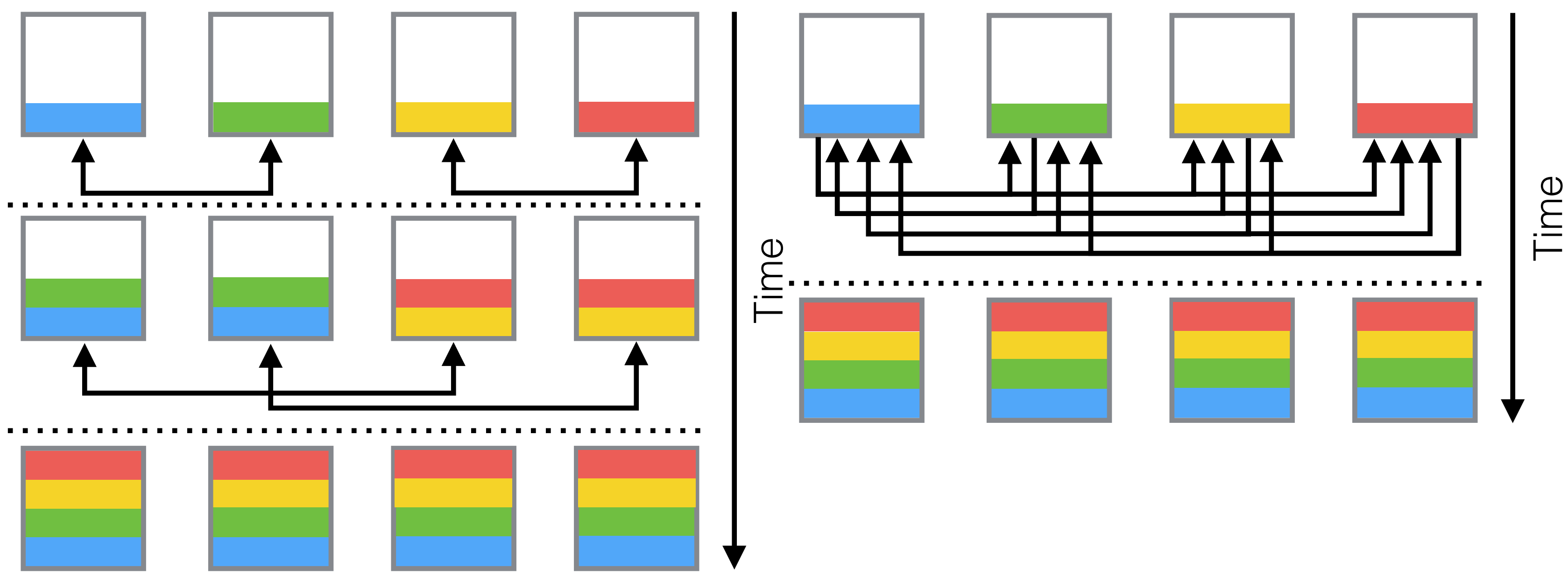
$$\alpha_p + b(\alpha_r + \cancel{\beta n + \gamma n}) \quad n = 0$$

$$(\alpha_p + b\alpha_r)\boxed{\log_2} N \rightarrow b = 1$$



First Idea

Pairwise Exchange is an **All-to-All**



Applied recursively like recursive doubling

$$(\alpha_p + b\alpha_r)\log_2 N \quad \rightarrow \quad (\alpha_p + b\alpha_r)\log_{b+1} N$$

Recursive Multiplying Model

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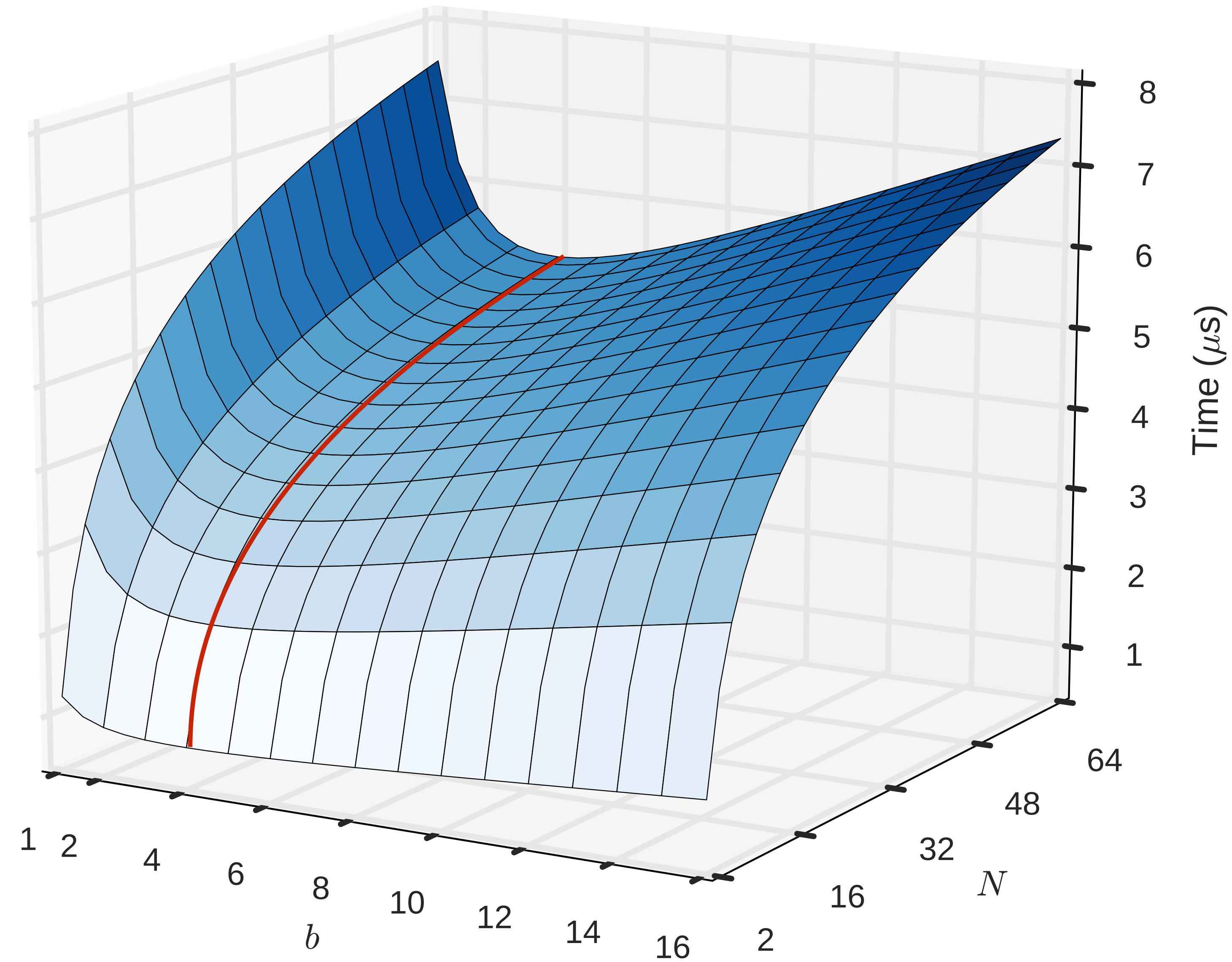
$$(\alpha_p + b\alpha_r)\log_{b+1} N$$

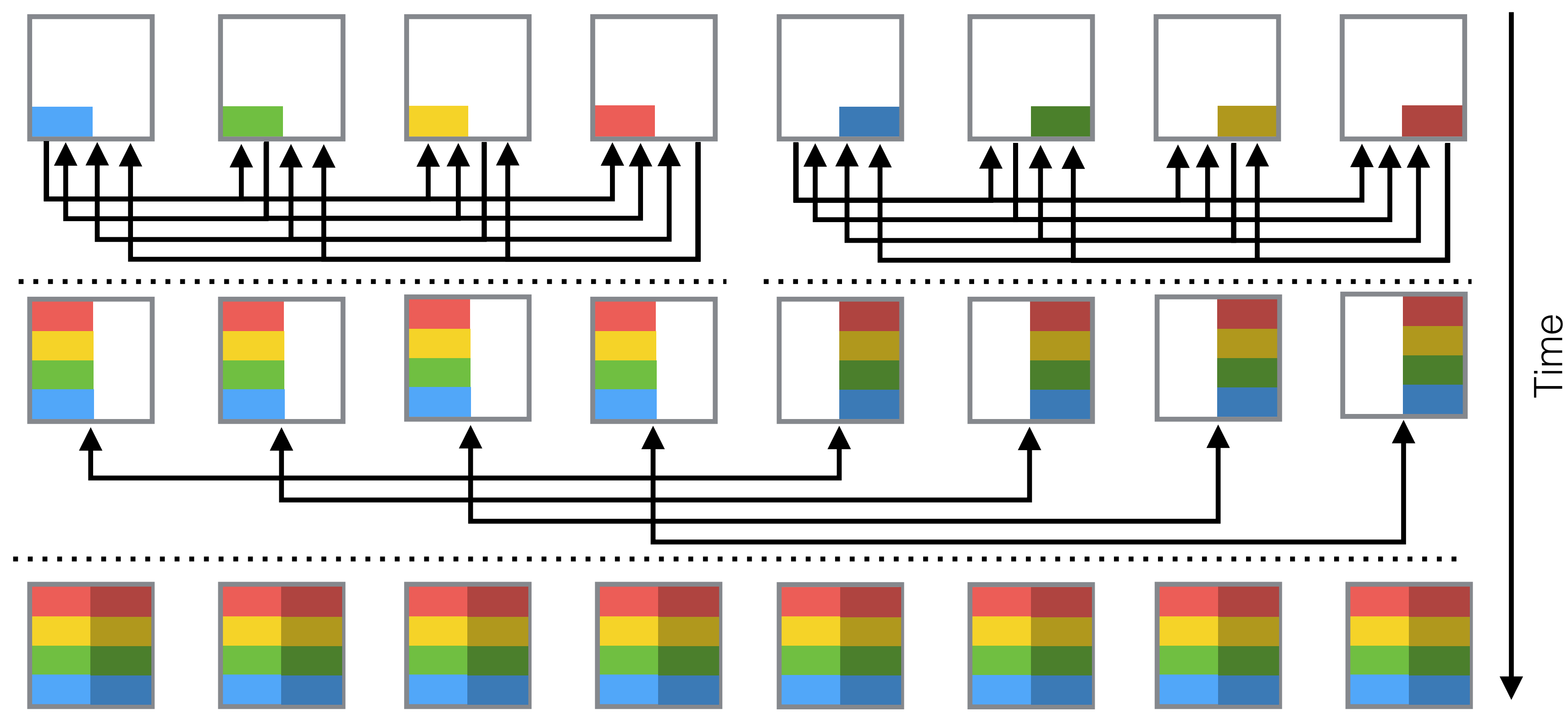
$$b_{opt} = e^{W\left(\frac{\alpha_p - \alpha_r}{e\alpha_r}\right)+1} - 1$$



N limited to powers of $b+1$

Second Idea

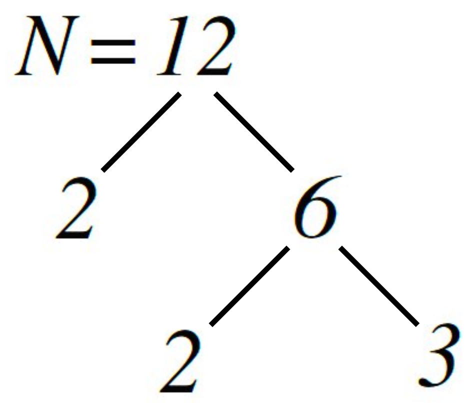
All-to-All **size can change**

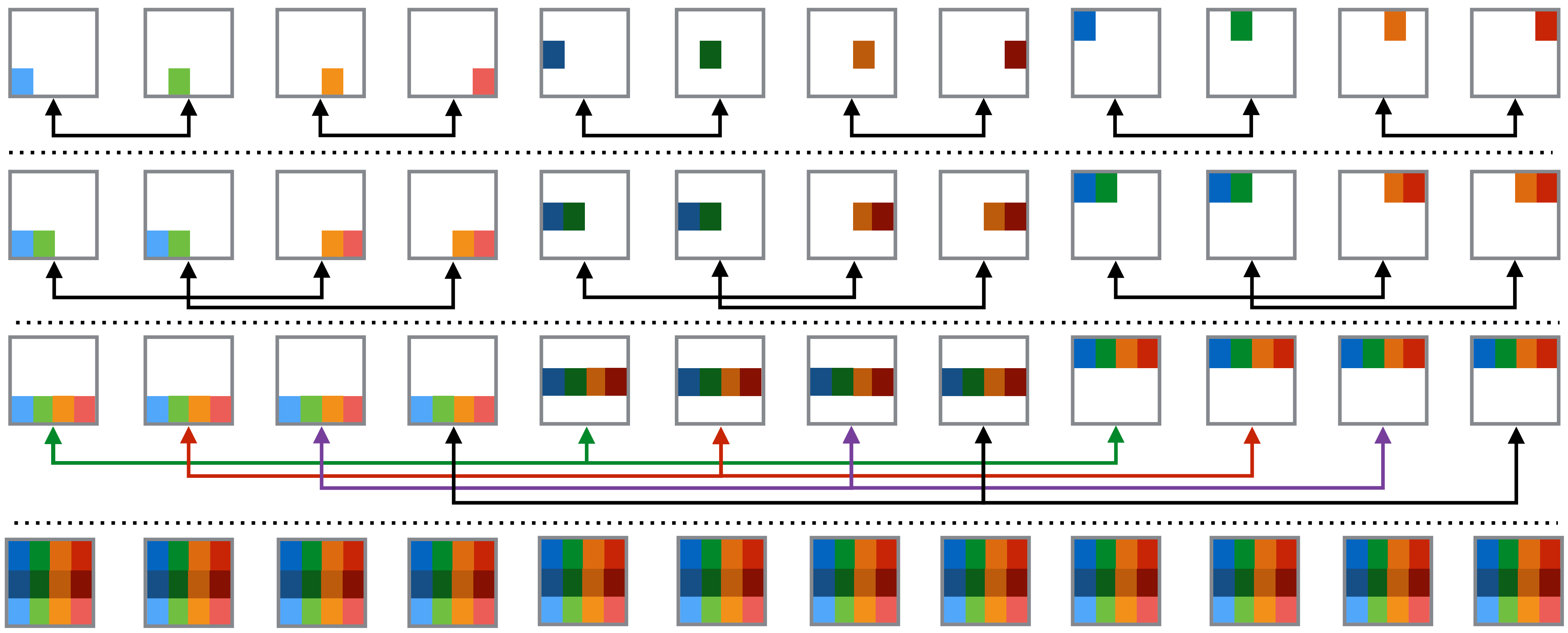


We have access to composite numbers instead of powers only

Recursive Multiplying

1. Prime factorisation of collective size

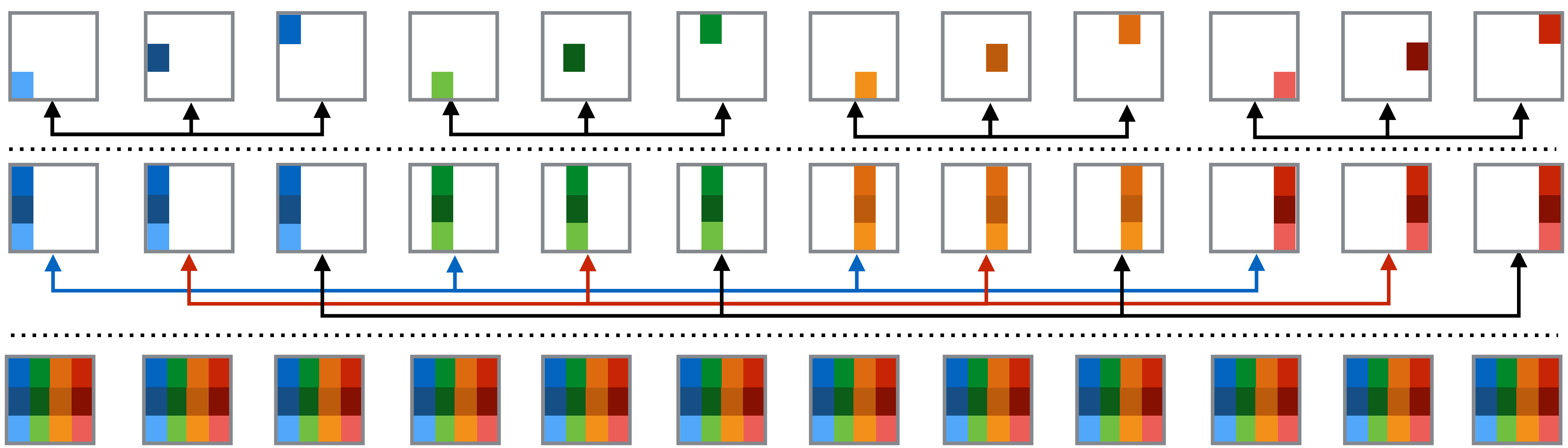




Recursive Multiplying

2. Aggregate factors
    - **Optimal** multicast usage
    - Dependent only on **overlap ratio**

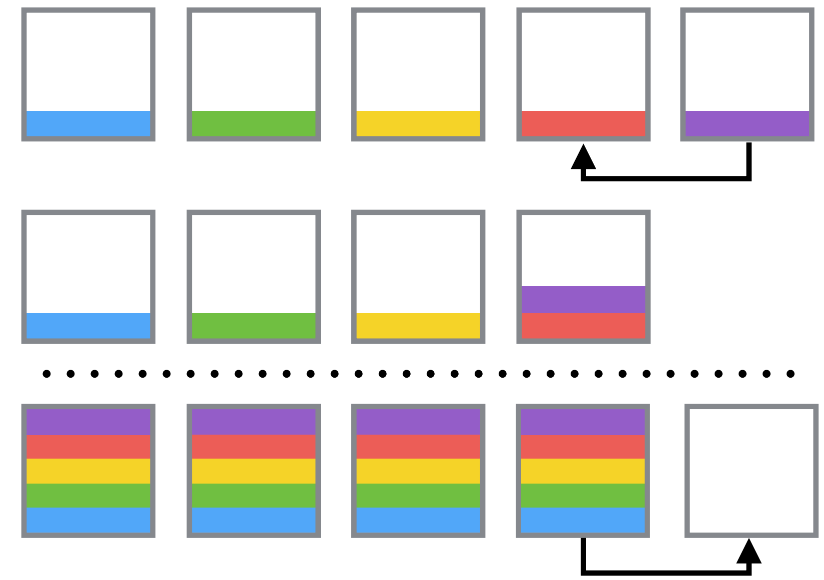
$$\left\{ \begin{array}{l} 12 \\ \boxed{3,4} \\ 2,6 \\ 2,2,3 \end{array} \right.$$

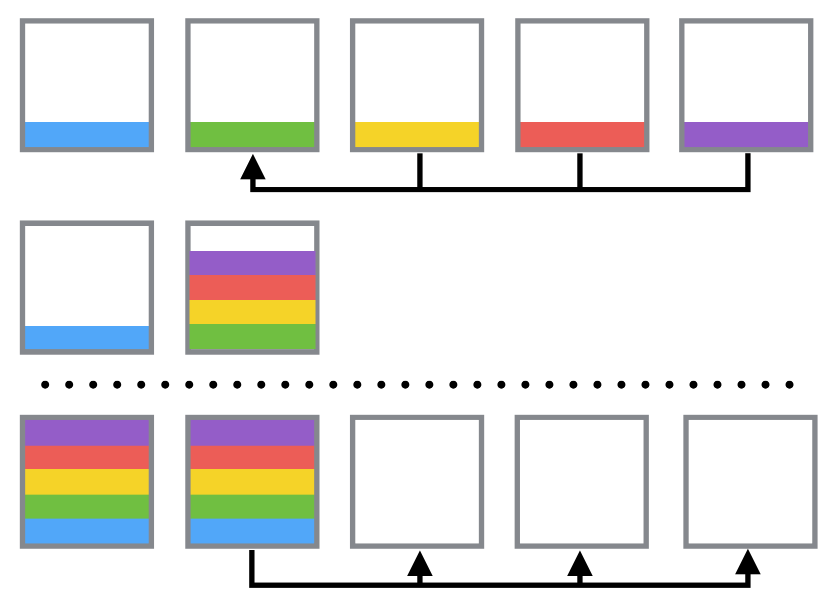


- power of 2 restriction -> large prime restriction

Splitting

- MPICH fix for the non-power-of-two case
- Make use of multicasting
- Generalised version of MPICH fix
- Still requires two additional stages



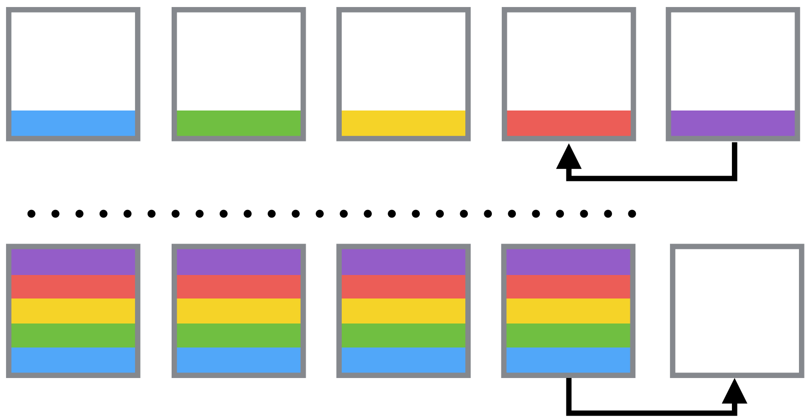


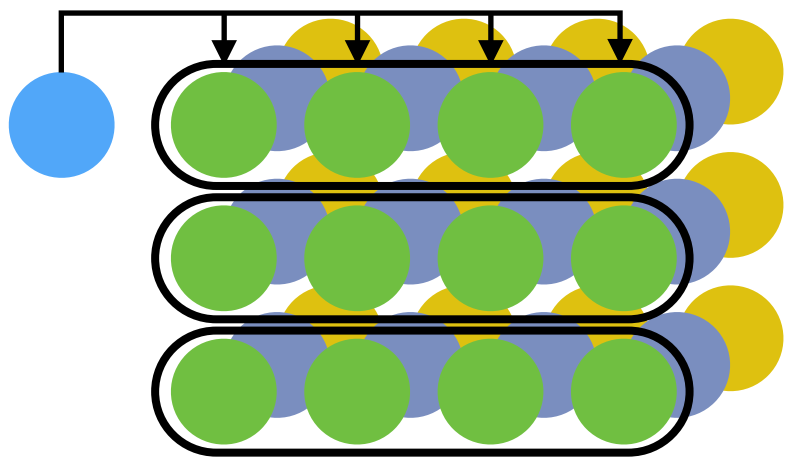
$\alpha_p + \alpha_r$

$\alpha_p + 3\alpha_r$

Merging

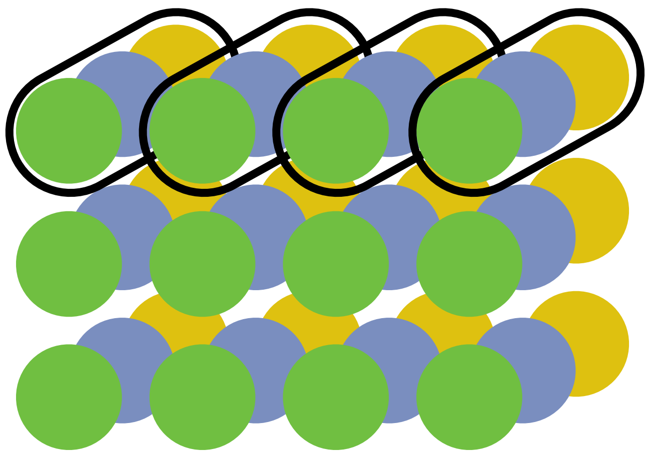
- Overlapping multiple patterns



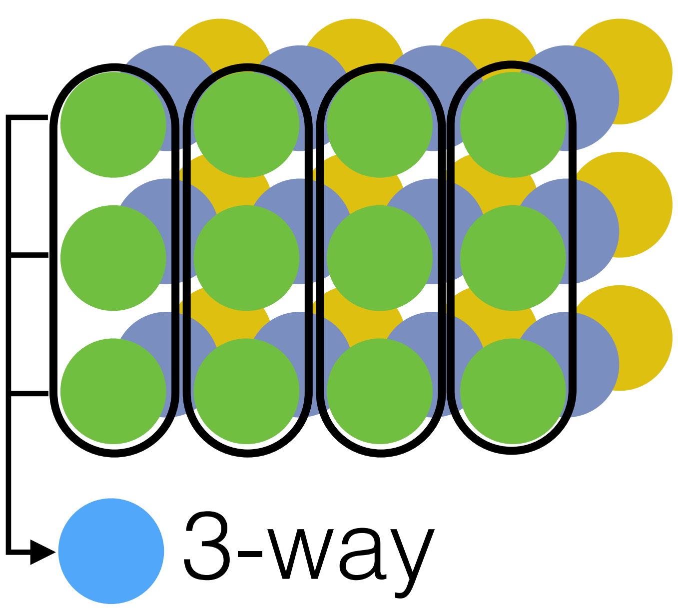


4-way

$$max(\alpha_p+4\alpha_r,\ \alpha_p+3\alpha_r)$$



3-way



3-way

$$\alpha_p+3\alpha_r$$

Results

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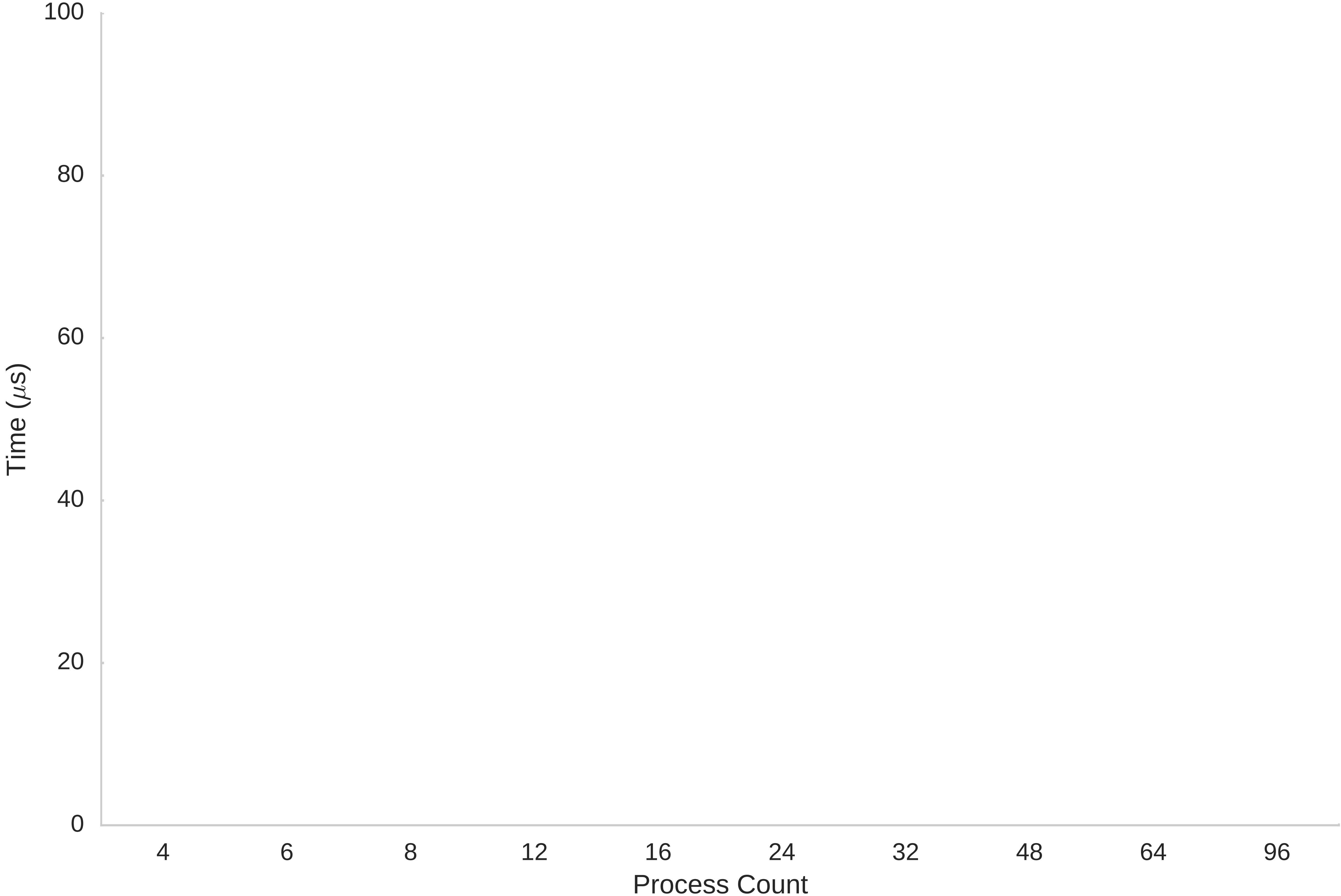

Results

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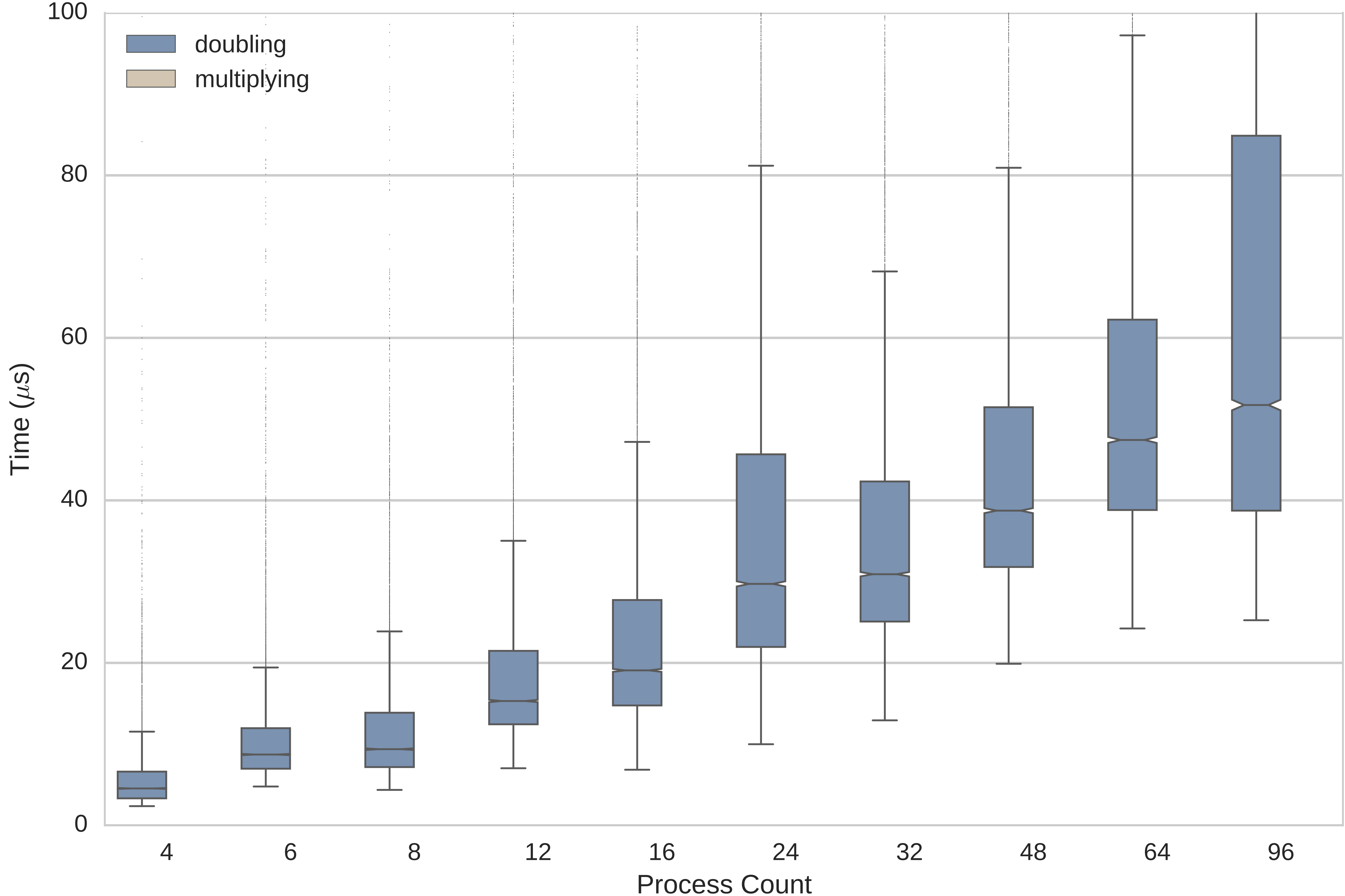

Results

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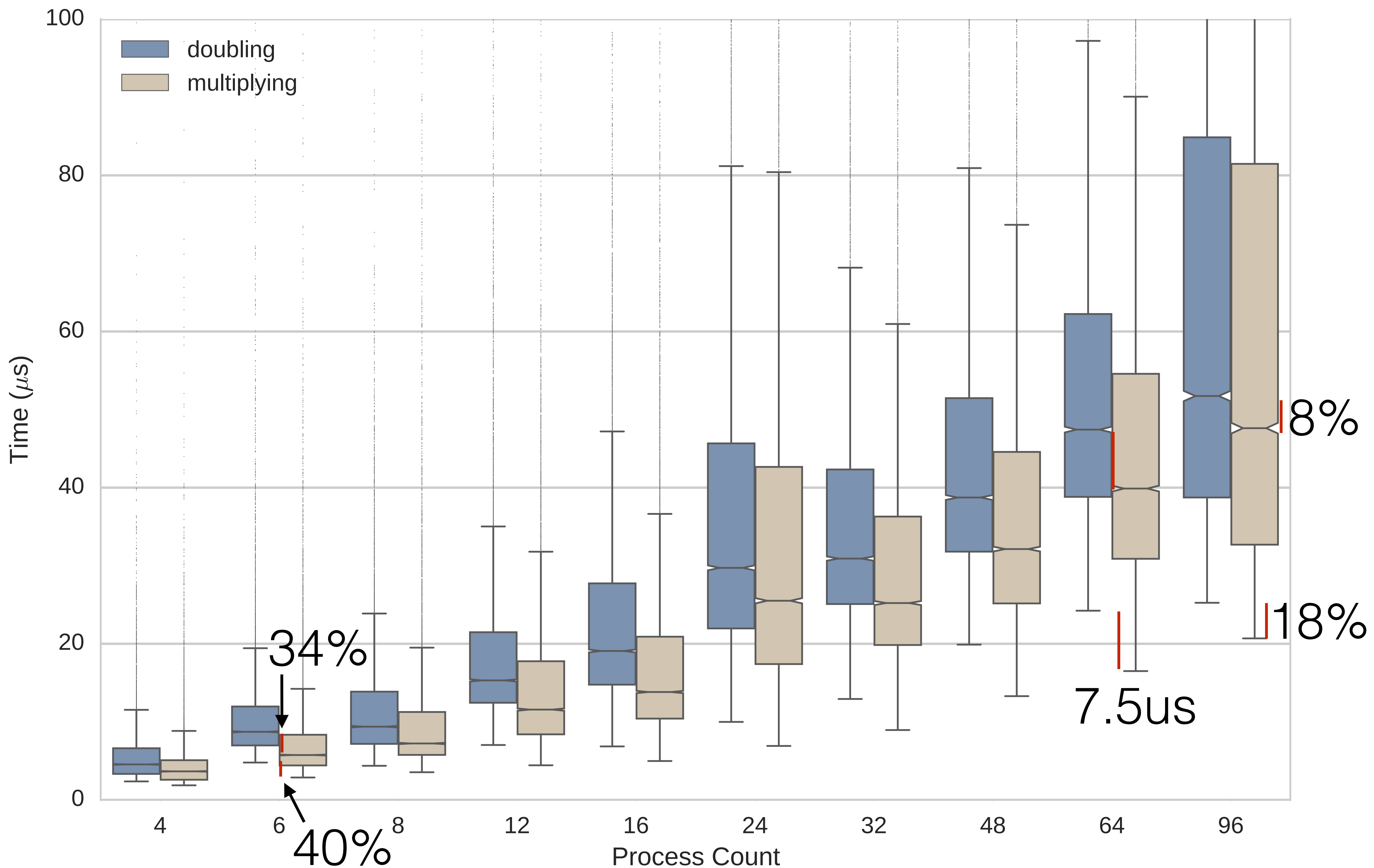

Summary

- Execution time less than recursive doubling consistently
- Drop in replacement for small message recursive doubling
- More pipelining and bandwidth